Technical Notes

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An Unsteady Laminar Boundary Layer with Separation and Reattachment

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Introduction

DURING the last few years, a controversy has developed in the literature on unsteady viscous flow concerning the existence of a singularity in regions of reversed flow.[1,2] Two examples, supported by analytic arguments, have been given in which a singularity appears. Williams and Johnson[3] have found that, for a semisimilar boundary layer with a mainstream resembling the Howarth distribution, a singularity, which appears to be of the Brown[4] type, occurs at *all* times and is centered at an internal point of the boundary layer. Bodonyi and Stewartson[5] have found that the boundary layer on a rotating disk in a counterrotating fluid develops a singularity on the axis of rotation after a finite time has elapsed from the onset of the unsteadiness, and presented an analytic argument in favor of the breakdown.

An important question is whether, for a two-dimensional boundary layer without spin, a singularity can develop after a finite time. Specifically, suppose that, in the usual notation

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0 \tag{1a}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\partial u_e}{\partial t}+u_e\frac{\partial u_e}{\partial x}+\frac{\partial^2 u}{\partial y^2} \tag{1b}$$

where

$$u=v=0 \text{ at } y=0 \qquad u\to u_e(x,t) \text{ as } y\to\infty \tag{1c}$$

and u satisfies given conditions at $t=0$ and at $x=0$. Then we ask: Does this problem have a solution for all time? Putting it in another way: Is it possible to choose u_e and the initial conditions so that at some t_o if a flow reversal occurs across the boundary layer, do solutions exist beyond $t>t_o$ without the appearance of a singularity? A definite answer would be very useful in the application of boundary-layer theory to the problem of dynamic stall[6] as well as to other problems in unsteady flows.

The protagonists of the singularity hypothesis believe that one can occur downstream of the onset of reversed flow, and three numerical solutions are quoted in justification, although none of them has any analytic argument in support. For one of these,[7] in which $u_e=2\sin x$, corresponding to the impulsive motion of a circular cylinder, a subsequent calculation of Cebeci[8] shows that the "singularity" is, in fact, a creature of the numerical method, and the correct solution is smooth for all values of t at which it was computed. The termination of his study was caused not by the appearance of a singularity, but by the rapid increase of displacement thickness as predicted by Proudman and Johnson.[9]

In this Note we are interested in the second example of Patel and Nash[10] who took

$$u_e=0 \text{ for } t\leq 0 \qquad \text{all } x>0$$

$$u_e=u_o\left(1-\frac{\omega t x}{c}\right) \text{ for } t>0, \qquad 0<x<x_o$$

$$u_e=u_o\left[1-\frac{\omega t}{c}(x_1-x)\right] \text{ for } t>0, \qquad x_o<x<x_1 \tag{2}$$

and a Blasius profile at $x=0$ for all t and at $t=0$ for all x. This mainstream velocity is actually continuous, but the pressure gradient is discontinuous at $x=x_o$, which causes numerical complications[11]; in addition, Patel and Nash[10] study a turbulent form of Eq. (1). The singularity they find is in the reversed flow region with $x<x_o$. Thus, if $\omega c/u_o=0.175$, $x_o=0.714$, it first appears at $x\simeq 0.6$ when $\omega t\simeq 1$.

We believe that the existence of the singularity should first be established for laminar flows before embarking on a study of the effects of turbulence, and so to test the hypothesis we choose

$$u_e=u_o[1-\alpha(x-x^2)(t^2-t^3)] \qquad 0<x<1 \qquad t>0 \tag{3}$$

with α being a positive constant and the other conditions being the same. The absence of the discontinuity in $\partial u_e/\partial x$ is not significant in this comparative study since the singularity occurs upstream of it. The absence of a linear term in t is simply to avoid the need for a double-structured solution near $t=0$ (see Ref. 11) and to enable us to perform the calculations more easily with the reversed flow.

Results

The solution of the system given by Eq. (1) is obtained by using Keller's efficient and accurate Box method (see for example, Ref. 12). A computer program recently developed by Cebeci and Carr[13] was used for this purpose. This program, which is applicable to both laminar and turbulent flows with no backflow, was modified to allow the calculation of flows with backflow. This was done by incorporating the zig-zag procedure described in Ref. 8. The details will be presented in a forthcoming report by Cebeci and Carr.

Figure 1 shows the computed local skin-friction coefficient in $0<x<1$ for various values of t and Fig. 2 shows the corresponding results for displacement thickness, taking $\alpha=20$ in each case. It is clear that the solution remains smooth for all calculated values of t, even when the region of reversed flow occupies the majority of the boundary layer and there is no hint of a singularity. Instead, we see the familiar rapid Proudman-Johnson growth[9] of the boundary layer in the reversed flow region which, unless special measures are taken, terminates the calculation due to the rapid thickening of the boundary layer. It is interesting to note that although $u_e<1$ in $0<x<1$, the favorable pressure gradient when x is near unity, more than compensates for the adverse gradient near $x=0$, and in the final state at $x=1$, the boundary layer is thinner than it would have been if $u_e\equiv 1$. In our opinion, there

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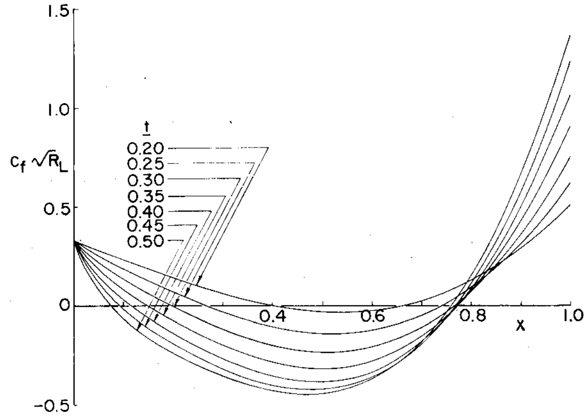

Fig. 1 Variation of local skin-friction coefficient $c_f\sqrt{R_L}$ with x for various values of t. Here $R_L = u_o L/\nu$ with u_o and L denoting a reference velocity and length, respectively.

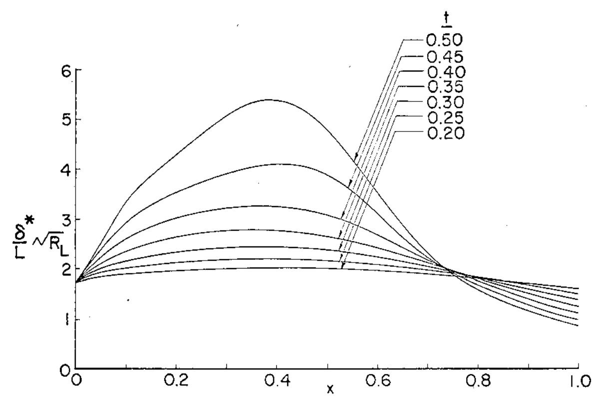

Fig. 2 Variation of dimensions displacement thickness $\delta^*/L\sqrt{R_L}$ with x for various values of t.

is no doubt that were an accurate solution of Eq. (1) computed under conditions of Eq. (2), no singularity would be found either and we believe that those found for the turbulent flows are spurious.

In the third cited example of a singularity,[14] the choice of u_e made the numerical work very difficult, but we maintain that an accurate solution would also be free of singularities for all t. Noting that Walker[15] has also given an example of an unsteady boundary layer with rapid growth of a reversed flow region but no singularity at finite t, we conclude that the answer to the question posed in the introduction is NO!; a singularity cannot develop in Eq. (1) at a finite time if the solution is free from singularities at earlier times.

Acknowledgment

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Reynolds Number Influence on Leeside Flowfields

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Introduction

LEESIDE flowfields have been of increased interest in the planning and development of supersonic aircraft and hypersonic space shuttle configurations. Although numerous reports have been published, very few systematic investigations are known. In this respect, it seems reasonable to consider first simple shapes like flat delta wings. The results of Refs. 1 and 2 provided a starting point for the author to derive a general description of the different types of flow and their boundaries.[3] In discussing these boundaries it is convenient to use the flow components normal to the leading edge, i.e., α_N and M_N, thus eliminating the sweep angle.

In Fig. 1 the following types of flow have been defined[3]: 1) leading-edge separation, 2) separation with embedded shock, and 3) shock-induced separation. Within the $\alpha_N - M_N$ diagram there exist boundaries, such as the well-known Stanbrook-Squire boundary, over which a continuous change over from one type of flow to the other occurs.

The main variables for the leeside flow over flat and slender delta wings with straight and sharp leading edges are 1) Mach number M_∞, 2) angle of attack α, 3) leading-edge sweep angle Λ, and 4) Reynolds number Re. While account is taken for the first three parameters, the question of Re influence on the flowfield remains to be answered. At hypersonic speeds quite a few investigations on that subject have been carried out (as examples see Refs. 4-6), while on the other hand at supersonic Mach numbers very little is known.[7,8]

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